Using graphs to give estimates

Work in pairs to solve the following problems.

1) Use the graph the function $f(x)=2^x$ (you find it in the next page) to give an estimate of the following numbers: if c is the number, find two numbers a and b such that $a<c<b$.

   a) $2^{\frac{3}{5}}$ b) $2^{\frac{-3}{2}}$ c) $2^{\frac{1}{3}}$ d) $2^{-\sqrt{2}}$ e) $2^{\sqrt{5}}$

   Then check the correctness using a calculator.

2) For each of the following values c, consider the number x such that $2^x=c$. Using the same graph, give an estimate of each value x.

   a) 3 b) $\frac{3}{2}$ c) $\frac{3}{4}$

   Then check the correctness using a calculator.

3) $3^{\sqrt{2}}$ is an irrational number. We want to find its initial five decimal digits using the following procedure.

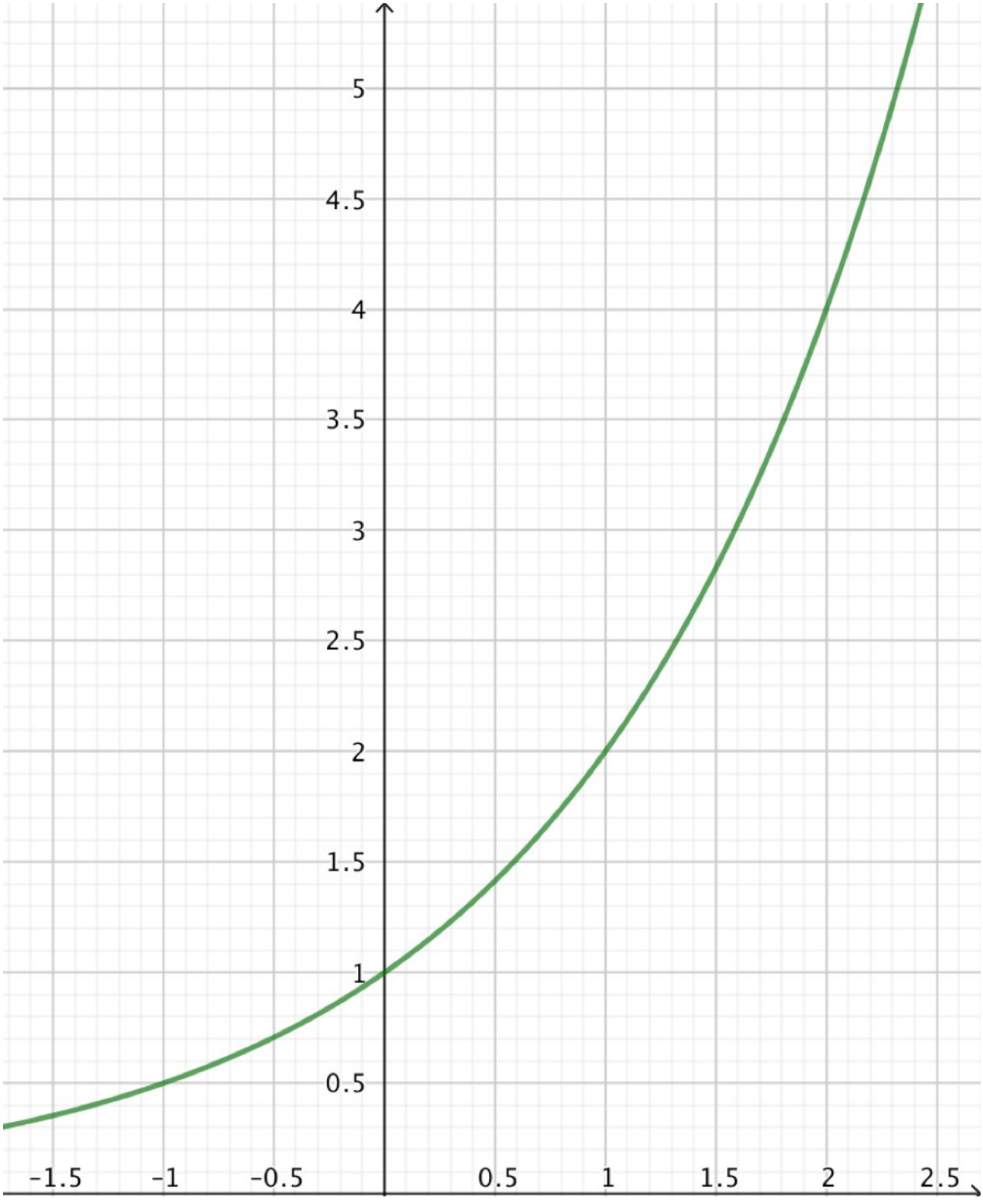
   a) Note that the exponent is $\sqrt{2}=1{,}414213\ldots$ (you can find this approximation using a calculator). Consider two sequences of numbers $a_0, a_1, a_2, \ldots, a_n, \ldots$ and $b_0, b_1, b_2, \ldots, b_n, \ldots$ that approximate $\sqrt{2}$ "from below" and "from above", as in the table. Complete the first two columns.

a_n	b_n	3^{a_n}	3^{b_n}
1	2	3	
1,4	1,5	4,65553672	5,19615242
1,41	1,42		
1,414			
	1,4143		
1,414213			

   b) Using a calculator compute the powers and complete the last two columns.

   c) In the last row the values of 3^{a_n} and 3^{b_n} have the initial five decimal digits equal. Can you now answer the initial question?

   d) How would you proceed if you wanted to know the initial six decimal digits of $3^{\sqrt{2}}$?



This activity was proposed by Luciano Cappello and Sandro Innocenti.