

An example of exponential decrease: Radioactive decay

Uranium 239 is an unstable isotope of uranium that decays rapidly: this means that it loses some of its mass because of radioactivity. We want to determine the rate of decay. We place 10 grams of U^{239} in a container and measure the amount remaining at 1-minute intervals. The table records the amounts.

| t = time in minutes | A(t) = grams remaining at time t |
|---------------------|----------------------------------|
| 0 | 10.00 |
| 1 | 9.71 |
| 2 | 9.43 |
| 3 | 9.16 |
| 4 | 8.89 |
| 5 | 8.63 |

Round all your answers to two decimal places.

- a) Show that these are exponential data.

Hint: use the characteristic property of exponential functions, that is compute $9.71/10.00$, $9.43/9.71$ etc. Are the results constant?

- b) Find A(6) that is, A at t=6 minutes.
- c) What is the percentage decay rate each minute? Explain in natural language what this number means.
- d) * *Hard for the moment, optional* * Find A at t = 30 seconds.
- e) Write an exponential function to express A as a function of t (measured in minutes).
- f) When is A = 6.50 g? You can use a spreadsheet to extend the table up to the desired row.
- g) The half-life of a radioactive substance is the time required for a quantity to reduce to half of its initial value. What is the half-life of U^{239} ? Use the table as in f).

Solutions

- a) because the ratio between A(t) and A(t-1) is always about 0.971 b) 8.38 g c) 2,9% d) 9.85
- e) $A(t)=0,971^t$ f) between 14 and 15 minutes g) between 23 and 24 minutes