

## Homework

### Exercise 1.

Fill in the gaps using the following words or numbers (some are used twice):

21 · fast · n · base · large · 1 · grows · rate  
2 · power · exponent · trillion · 4 · doubles

In the legend, the number of rice grains \_\_\_\_\_ at each square. This means that:

- On square 1 there is just \_\_\_\_\_ grain;
- On square 2 there are \_\_\_\_\_ grains;
- On square 3 there are \_\_\_\_\_ grains;
- On square \_\_\_\_ there are 8 grains;
- On square \_\_\_\_ there are  $2^{n-1}$  grains.

The variable is in the \_\_\_\_\_: for each number  $n$  we consider the \_\_\_\_\_  $2^n$  (this reads: “2 to the  $n$ -th power” or “2 to the power  $n$ ”). Then we say that the number of grains \_\_\_\_\_ exponentially. Note that this is different from taking powers of  $n$ , like  $n^2$  or  $n^3$ , because here the variable is in the \_\_\_\_\_. These kinds of growth are called polynomial.

How fast does the power  $2^n$  grow?

Here we are considering the natural powers of 2: the \_\_\_\_\_ of growth is given by the \_\_\_\_\_ of the power, which in our case is 2.

Exponential growth is really \_\_\_\_\_: if the base is 2 at each step the number of grains \_\_\_\_\_ and after a few squares we need a very \_\_\_\_\_ number of grains.

For example, to get more than one million grains on one square you need just \_\_\_\_\_ squares; to get more than one \_\_\_\_\_ grains on one square you need just 31 squares.

### Exercise 2.

In question 3 in class you completed a table. Consider a slightly different table:

|        |   |   |   |   |   |   |   |   |   |    |    |    |    |     |     |       |       |
|--------|---|---|---|---|---|---|---|---|---|----|----|----|----|-----|-----|-------|-------|
| Square | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | ... | $n$ | $n+1$ | $n+2$ |
| Grains | 0 | 1 | 2 | 4 |   |   |   |   |   |    |    |    |    |     |     |       |       |

- a) Complete the table. Find rules to describe how to move in the columns (from the top row to the bottom row) and in the rows (from left to right in each row):
- In the top row moving right means to add 1 to the number in the square.

- In the bottom row...
  - Moving vertically from the top row to the bottom row means ...
- b) Can you give a general rule for the growth between square  $n$  and square  $n+1$ ?
- c) Can you give a general rule for the growth between square  $n$  and square  $n+k$ ? Explain it!  
Hint: you can extend and use the table above. Recall the properties of the powers!

### Exercise 3.

Watch the video at: <https://www.youtube.com/watch?v=gEwzDydcIWc>. It shows a microbiological culture of the bacteria *Escherichia Coli*. Script:

“Bacteria reproduce very simply and rapidly by doubling their contents and splitting in two. Just one bacterium dividing every 20 minutes could produce nearly five thousand billion billion bacteria in one day.”

Write down the number using scientific notation. Assuming that the speed of duplication is realistic, is the estimate given in the video correct? Show your computations.

### And, in case you are curious... A note on numbers<sup>1</sup>

In Europe comma (,) is used as a decimal separator in numbers. For example we write “I paid 21,34€”. We use a point or a space to separate thousands, as in: “Last year I saved 12'500 €” or “Last year I saved 12.500 €” or “Last year I saved 12 500 €”.

People in the US, UK and other English speaking countries use different separators: they write “I paid 21.34 \$” and “Last year I saved 2,500 \$”.

In scientific contests, the International System of Units (SI) recommends to use a point or a comma for the decimal separator and a blank space or no symbol at all for the thousand separator.

Correct: 101 000,330 245 and 101 000.330 245

Correct: 101000,330245 and 101000.330245

Wrong: ~~101,000.330,245~~ and ~~101.000,330.245~~

However, for “very large” or “very small” numbers you can always use scientific notation.

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<sup>1</sup> You can read more about this at <http://www.languageediting.com/format-numbers-eu-vs-us/>