The game of chess

1. Watch the video https://www.youtube.com/watch?v=eJmWu18pWtI up to 1:15. Complete the script:

"Ambalappuzha is a small town in southern ___________ which is __________ for two things: the Sri Krishna Temple and Paal Payasam. Paal Payasam is a dessert that's made mostly of sweetened boiled ____________ and ____________.

According to legend the _______________ of Ambalappuzha was a big chess enthusiast and he often *challenged** visitors to a ___________.

One day a traveling *sage** arrived at the court of the king and ____________ him to a game of ___________. The king accepted and asked the __________ what *he'd like** as a prize if he won.

The sage said he'd like some ____________: he'd like ___________ grain of rice on the ___________ square of the *chessboard** and for the number to be *doubled** for each ________________ after that.

So the king put a single _____________ of rice on the first square, _____________ grains on the second square, __________ on the third and ___________ on the fourth. He looked at the grains and thought: "What a ridiculously modest prize!". He accepted the sage's wish and the game _____________."

2. Why does the king think that the prize is "ridiculously modest"? Do you think so, too? Why? Do you think the prize will be more than 100 kg rice? More than one *ton** rice?

3. How many grains are there on each of the following squares of the chessboard? Complete the table and then explain the rule in English.

Square	1	2	3	4	5	6	7	8	9	10	12	20	30	...	n
Grains	1	2													

4. How many grains are there on the last square of the chessboard? Write this number as a power of 2. Then give an estimate using scientific notation. Hint: approximate 2^{10} using a power of 10.

5. On which square does the number of grains *exceed** 1,000,000? 500,000,000? 500,000,000,000?

6. Watch the video up to 2:20 and check your answers. Complete the script:

"The sage __________ well and easily won. True to his word the king ______________ a bag of rice to be brought and ________________ to count out the grains onto the squares. He was _______________ when he got to just the 12th square and found that he needed to put ____________ grains onto it.

He realized that the ______________ was going to be a lot ______________ than it at first seemed. After a long _______________ he *figured out** that for the 30th square he'd need about 536 *million** grains, by the 40th square he'd need about 550 *billion** grains of rice and for the ________________ square, the 64th, he'd have to count out a *staggering** 9 million 220,000 *trillion** grains.

Altogether he'd have to give the sage about 18 million 500,000 trillion grains."

It is not easy to compare these numbers. Let's try to get a sense of the growth.

7. Suppose a single grain of rice weighs 0.029 g. What is the weight of the grains on square 64? Give an answer in *tons**.

8. What is the total weight on the chessboard? Hint: note that $2^0+2^1+2^2+...+2^n=2^{n+1}-1$

9. Watch the video up to 2:52. Complete the script and check your answers to 7. and 8.:

“A ______________ long grain of rice weighs on *average** about 0.029 grams, so the ____________ weight of rice that the king needed to ______________ the sage was about 535 billion tons. That's ______________ than a thousand times greater than all the world's milled rice __________________ in 2019-20.

The king realized it was __________________ for him to pay his debt.”

10. The story continues up to 3:20. Complete the script:

“In the ______________ the sage turns out to be the god Krishna in *disguise**. When he saw how upset the king was, he appeared to the king in his ____________ form. He told the king that he didn't have to *settle the debt** immediately but could _______________ it over time. So the king started to serve Paal Payasam in the temple free every day to pilgrims until the _______________ was cleared.”

11. Make <u>a graph of the number of grains as a function of the number of squares</u>. How many points of the graph fit in a A4 paper (the size of your notebook) if you use 0.5 cm as 1 rice grain?

12. Let *x* be the number of squares of the chessboard. Write a function describing the number of grains of rice per square.

13. Watch the video to the end. Underline the mathematics words in the following script**:**

“In mathematics this kind of growth is described as exponential and as we've seen it can give rise to large numbers very quickly. The function describing the number of grains of rice per square is 2^{x-1} (“2 to the x minus 1”) where x is the number of squares. And you can see from the graph how *steeply** it grows.

So remember the legend of Ambalappuzha and if you're ever in town don't forget to visit the temple for your free bowl of rice.”

14. The graph shown in the video is not correct. Can you say why? Compare it with yours.

Some vocabulary
to challenge = to invite in a competition, to make a race and see who wins
sage = wiseman, a wise and smart person
he’d like = he would like: conditional sentence
chessboard = where you play chess, an 8-by-8 table
to double = to make it twice as much, e.g. when you double 10 you get 20
ton = a unit of measurement corresponding to 1,000 kg
to exceed something = to pass something, to go beyond something, to become more than something
to figure out = to understand, to realize
million = 1,000,000 = 10^6
billion = 1,000,000,000 = 10^9
trillion = 1,000,000,000,000 = 10^{12}
staggering = disconcerting, astonishing, incredible
average = mean; example: “I got 8 and 6 in mathematics, so my average is 7.”
disguise = camouflage, dressed strangely in order not to be recognized
to settle a debt = to pay back all the money
steeply = growing quickly; example: “That mountain climb is very steep”.

Some answers

4. Since $2^{10} \approx 10^3$ on the 64th square there are 2^{63} grains, that is about $8 \cdot 10^{18}$.
5. See the video.
7. About $2.7 \cdot 10^{17}$ g = $2.7 \cdot 10^{11}$ tons
8. About $5.4 \cdot 10^{11}$ tons
12. $f(x) = 2^{x-1}$
14. Look at the first part of the graph in the video, when x is small…