CLIL Module Plan

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School	Liceo da Vinci - Trento			
School Grade	○ Primary		○ Middle	◉ High
School Year	○ 1	○ 2	○ 3	◉ 4	○ 5
Subject	Matematica	**Topic**	Combinatorics, probability.	
CLIL Language	◉ English		○ Deutsch	

Personal and social-cultural preconditions of all people involved	INSTITUTIONAL FRAMEWORK CONDITIONS The scientific high school "Leonardo da Vinci" is one of the historical "Liceo" of the Province of Trento. Nowadays the high school proposes two curricula, the ordinary scientific curriculum and the applied sciences scientific curriculum. ANTHROPOGENIC AND SOCIO-CULTURAL FACTORS OF THE GROUP OF LEARNERS The class consists of 23 students. There are no SEN students or students of foreign origin. LEARNING PRECONDITIONS The large part of the lessons of the present CLIL module takes place in the classroom. This is equipped with a PC, an interactive whiteboard (IWB), a large blackboard, and a projector. A few activities are carried out in the ICT lab, where each student can use a PC for calculations, data collection, and data analysis. The academic performance of the class for what concerns the scientific subjects is average. The behavior of the students is very polite although the level of participation is not high. The majority of the students are highly motivated and willing to learn new concepts, however there are elements of the class that do not have a specific interest in the subject or feel the foreign language as a barrier. TEACHER PROFILE The teacher Giovanni Lombardi (T), teaches Mathematics and Physics in various classes, ranging from the 1st to the 4th grade of the school and for this CLIL module has the role of main teacher. CEFR level: C1 (IELTS certification dated 18/01/2018). The Mathematics teacher in this class does not have a sufficient level of spoken English to actively participate to the lessons. However, she will participate to the lessons listening to the students, observing them, encouraging them to participate more actively when needed. STUDENT GROUP PROFILE All the students are Italian mother tongue, and their average CEFR level is B1+, but a few have a C1 certification. The students have limited CLIL experience, as they have followed a total of 30h of CLIL lessons during the previous school year.

Students' prior knowledge, skills, competencies	Subject	Language
	Students have a solid knowledge of the basic maths operations in the field of real numbers. In the first year they have studied set theory and are able to work with sets and to carry out the basics operations between sets (union, intersection, ...) Students are able to solve first and second degree equations in one variable and systems of equations.	The students have adequate communication skills. They can interact both with the teacher and with their fellow students in English, but they do not have a specific knowledge of the scientific terms necessary to describe the physical phenomena considered in this CLIL module. Students have good reading and writing skills

Timetable fit	◉ Module	Length 20 lessons of 50' each

Description of teaching and learning strategies	• The learning and teaching objectives are disciplinary-specific, interdisciplinary,and communicative. The lessons are designed to encourage the development of problem solving skills, critical thinking, creative thought, collaboration, communication, and time managing. • The methodological approaches are various, in order to meet different learning styles and to promote the development of different skills: interactive lessons, group work (especially when the task is complex), pair work, individual work, TPS (Think, Pair, Share), cooperative learning. During the "student-centered" activities the teacher acts as facilitator and guide. • Interaction and communication are promoted as much as possible by the teachers,(by asking questions and inviting the students to comment or express their ideas) and by activities focused on group or pair work. During these activities, the teacher circulates and models language, concepts and cognition. • A variety of online resources, including online lessons, spreadsheets, and practice material is used. The main sources of this online material are Khan-Academy and Youtube. • The teacher provides different materials to support content and language scaffolding, and to consolidate learning, e.g. worksheets, extra exercises, and homework. • During most activities a formative assessment by the teachers is provided and peer- or self-evaluation are encouraged. At the end of the Module a summative assessment is provided.

Overall Module Plan

Unit: 1 Combinatorics and probability **Unit length:** 10 lessons of 100' each	**Lesson 1** Permutations
	Lesson 2 Permutations and factorials
	Lesson 3 Combinations and binomial coefficients
	Lesson 4 Probability - Classical definition
	Lesson 5 Probability - Union
	Lesson 6 Probability - Conditional probability
	Lesson 7 Probability - Independent events
	Lesson 8 Probability - Total probability
	Lesson 9 Probability - Frequentist interpretation
	Lesson 10 Final test + discussion

CLIL Lesson Plan

Unit number	1	Lesson number	1	Title	Permutations

Activity	Timing	Learning Outcomes	Activity Procedure	Language	Interaction	Materials	Assessment
1	5	Having an overview of the activities that will be carried out during this module.	- T explains the lesson plan for the present unit and gives a brief overview of the activities. - Ss ask questions and clarifications.	**Skills** ▣ L ▣ S ▢ R ▢ W **Key vocabulary** Plan, experiment, theory, theoretical, teamwork, pair work, ICT, combinatorics **Communicative structures** Sentences about planning, e.g. "We/You are going to…"	▣ Whole class ☐ Group work ☐ Pair work ☐ Individual work	None.	None.
2	15	Making hypotheses. Improving own problem solving skills. Modelisation.	BRAINSTORMING. - T proposes the following problem: "we have two chairs labeled #1 and #2: in how many ways can	**Skills** ▣ L ▣ S ▢ R ▢ W **Key vocabulary** label, arrange, order.	▣ Whole class ☐ Group work ☐ Pair work	None.	Formative: T assesses the insight of the hypothesis made by Ss. T informally assesses the

			we arrange 4 Ss (named A, B, C, D) on the 2 chairs". - T stresses that a situation with S A in chair #1 and S B in chair #2 is to be considered different from the situation with S B in chair #1 and S A in chair #2. - T writes on the blackboard the main points of the hypotheses made by Ss. - To clarify the problem T can call 4 Ss to the blackboard and, using two chairs, simulate some of the configurations described by the problem. - Ss are encouraged to make hypotheses, to motivate them, and to illustrate them to the rest of the class.	**Communicative structures** Sentence structures related to mathematical relations, to making and motivating hypotheses, to agreeing/disagreeing, e.g. - in how many ways can we arrange... - I think that... - If ... sits in chair #1 and... then...	☐ Individual work		language used to formulate the hypotheses.

| 3 | 10 | Learning to use tree-diagrams to solve problems related to counting configurations. Reflecting on different strategies to solve a problem. Self-assessment. | - T shows how the problem of the previous activity can be easily solved using a tree diagram. - The result is written first as a product, i.e. number of ways = 4*3=12. - This will help Ss generalise the result later in the lesson. | **Skills**

 L S R W

 Key vocabulary
 tree-diagram, branch, node, fix

 Communicative structures
 Sentence structures related to mathematical relations, and to the description of a diagram, e.g. - three branches start/stem out of this node - the total number of ... | ☒ Whole class
 ☐ Group work
 ☐ Pair work
 ☐ Individual work | None. | Self-assessment: Ss can compare their version of the solution to a correct and powerful method. |

| 4 | 20 | Employing the knowledge acquired during the previous activities. Improving own problem solving skills. Presenting results to peers. Comparing own results with those of peers. Giving opinions/comments. | - T asks Ss to use the tree-diagram method to solve the same problem of activity 1 but with 4 Ss and 3chairs (instead of 2 as in the original problem). - Ss have 5 minutes to solve the problem individually. - Then Ss form pairs and discuss the solutions with their mate (for about 5 minutes). - The solution is finally discussed with the whole class. - A S can be asked to solve the problem for the whole class at the blackboard. | **Skills**

| L | S | R | W |

Key vocabulary
label, arrange, order, tree-diagram, branch, node, fix

Communicative structures
Sentence structures related to mathematical relations, to presenting results, to giving/receiving advice, to agreeing/disagreeing, e.g. - My result is … what is yours? - Could you explain how you did that? - I don't think that's correct. Try… instead. | ▣ Whole class
☐ Group work
▣ Pair work
▣ Individual work | None. | During the activity T goes around the class evaluating the level of participation and comprehension of Ss (asking targeted questions if necessary). |

| 5 | 10 | Learning to use tree-diagrams to solve problems related to counting configurations. Learning to solve a problem independently. Self-assessment. | - T asks Ss to use the tree-diagram method to solve the same problem of activity 1 but with 5 Ss and 3 chairs - Ss solve the problem individually. - A S is then chosen to solve the problem at the blackboard for the whole class. | **Skills** L S R W **Key vocabulary** label, arrange, order, tree-diagram, branch, node, fix. **Communicative structures** Sentence structures related to mathematical relations, and to the description of a diagram, e.g. - three branches start/stem out of this node. - the total number of ...is... | ☑ Whole class ☐ Group work ☐ Pair work ☑ Individual work | None. | During the activity T goes around the class evaluating the level of comprehension of Ss, giving advice if needed. Self-assessment: Ss can compare their version of the solution to the correct one. When the problem is solved at the blackboard T evaluates the solution given by S for both the content and the language (formative assessment.) |
| 6 | 20 | Generalising the results of specific problems. Making hypotheses, Critical thinking. Checking | - T asks Ss to find the general formula that describes the number of ways to | **Skills** L S R W | ☑ Whole class ☑ Group work | | During the activity T goes around the class evaluating the |

| | | the validity of an hypothesis. Cooperation in a group work. Self-assessment. | arrange n elements in k places. - Ss work in groups for 12 minutes. - Ss make hypothesis and discuss them with their group mates. - When 12 minutes expire T writes on the blackboard the solutions proposed by the different group. - The solutions are discussed with the whole class. - The correct solution is written in the form number of ways= D_n,k= n*(n-1)*...*(n-k+1). - The term permutation is introduced. | **Key vocabulary** label, arrange, order, tree-diagram, number, product, factor, element, permutation, integer (number).

 Communicative structures Sentence structures related to mathematical relations, to making hypotheses, to the description of a diagram, e.g. - the solution is a product of … factors - the number of factors is equal to the number of nodes - the number of permutations of n elements in k places is… | ☐ Pair work
 ☐ Individual work | | | level of participation and comprehension of Ss. Self-assessment: Ss can compare their version of the solution to the correct one. During the discussion of the solution T assesses the hypothesis made by Ss for insight and language |

| 7 | 12 | Understanding the main points of a short introductory video about counting outcomes with tree diagram. Identifying important data, information, and keywords. Vocabulary building. | - Ss watch a video about counting outcomes using tree-diagrams. During the video Ss take notes and ask questions. - At the end of the video Ss have some time to work in pairs on the vocabulary, trying to clarify the meaning of the keywords through discussion and comparison. | **Skills**

L S R **W**

Key vocabulary
label, arrange, order, tree-diagram, number, product, factor, element, permutation, car, engine, likely.

Communicative structures
Sentence structures related to mathematical relations, to making hypotheses, to the description of a diagram, e.g. - we have … equally likely possibilities | ▣ Whole class
☐ Group work
▣ Pair work
☐ Individual work | T shows the video "Counting outcomes using tree diagram": link (from Khan Academy). | None. |

| 8 | 8 | Applying the knowledge acquired during the present lesson. Improving own problem solving skills. | - T hands out an exercise sheet to Ss. - Ss solve exercises 1, 2, 3 individually. - Homework: solve exercise 5 and complete the Glossary given in file U1_L1_ALL1.pdf by adding the keywords noted down during the lesson. | **Skills**

| L | S | **R** | **W** |

Key vocabulary
see the "Glossary" section in file U1_L1_ALL1.pdf

Communicative structures
Sentence structures related to instructions,e.g. - calculate the number of... | ☐ Whole class
☐ Group work
☐ Pair work
▣ Individual work | • U1_L1_ALL1.pdf
• U1_L1_ALL2.zip

Each Ss receives a printed copy of the exercise sheet "Permutations.pdf" (file U1_L1_ALL1.pdf - editable version U1_L1_ALL2.zip). | During the activity T goes around the class evaluating the of comprehension of Ss, giving advice if needed. |

CLIL Lesson Plan

Unit number	1	Lesson number	2	Title	Permutations and factorials

Activity	Timing	Learning Outcomes	Activity Procedure	Language	Interaction	Materials	Assessment
1	10	Reviewing the results of the previous lesson. Consolidating the knowledge acquired in the previous lesson. Self-assessment.	- T briefly reviews the main concepts examined during the previous lesson. - Ss can ask questions about the contents of the previous lesson. - A S is chosen to solve the exercise given as homework for the class.	**Skills** L S R W **Key vocabulary** label, arrange, order, tree-diagram, number, product, factor, element, permutation, likely. **Communicative structures** Sentence structures related to mathematical relations, to making hypotheses, to the description of a diagram, e.g. - To solve the problem I used a tree-diagram...	☐ Whole class ☐ Group work ☐ Pair work ☐ Individual work	• U1_L1_ALL1.pdf	Formative: T assesses the solution of the problem for both content and language. Self-assessment: Ss compare their solutions to the correct ones.

| 2 | 15 | Employing the knowledge acquired during the previous activities to solve a problem that involves the permutations of n elements in n places. Improving own problem solving skills. Presenting results to peers. Comparing own results with those of peers. Giving opinions/comments. | - T asks Ss to use the tree-diagram method to solve the same problem of activity 1 or lesson 1 but with 4 Ss and 4 chairs. - Ss have 5 minutes to solve the problem individually. - Then Ss form pairs and discuss the solutions with their mate (for about 5 minutes). - The solution is finally discussed with the whole class. - A S can be asked to solve the problem for the whole class at the blackboard. | **Skills**

L S R W

Key vocabulary
label, arrange, order, tree-diagram, branch, node, fix

Communicative structures
Sentence structures related to mathematical relations, to presenting results, to giving/receiving advice, to agreeing/disagreeing, e.g. - My result is … what is yours? - Could you explain how you did that? - I don't think that's correct. Try… instead. | ☒ Whole class
☐ Group work
☒ Pair work
☒ Individual work | None. | During the activity T goes around the class evaluating the level of participation and comprehension of Ss (asking targeted questions if necessary). |

| 3 | 15 | Generalising the results of specific problems. Making hypotheses, Critical thinking. Checking the validity of a hypothesis. Cooperating. Learning the definition of factorial. | - T asks Ss to find the general formula that describes the number of ways to arrange n elements in n places. - Ss work in groups of 3/4 for 7 minutes. - Ss make hypotheses and discuss them with their group mates. - When 7 minutes expire T writes on the blackboard the solution proposed by the different group. - The solution are discussed with the whole class. - The correct solution is written in the form $P\_n = n*(n-1)*...*1=n!$. - The definition of factorial is introduced. | **Skills**

 L \| S \| R \| W

 Key vocabulary
 label, arrange, order, tree-diagram, number, product, factor, element, permutation, factorial, integer (number), exclamation mark.

 Communicative structures
 Sentence structures related to mathematical relations, to making hypotheses, to the description of a diagram, e.g. - the solution is a product of ... factors. - the number of permutations of n elements (in n places) is... - the factorial of a (positive) integer number is defined as... | ▣ Whole class
 ▣ Group work
 ☐ Pair work
 ☐ Individual work | None. | During the activity T goes around the class evaluating the level of participation and comprehension of Ss. Self-assessment: Ss can compare their version of the solution to the correct one. During the discussion of the solution T assesses the hypothesis made by Ss for insight and language |

| 4 | 15 | Understanding the main points of a short video about factorials. Learning the definition of 0! (0 factorial). Identifying important data, information, and keywords. Vocabulary building. | - Ss watch a video about the definition of 0!. During the video Ss take notes and ask questions. - At the end of the video Ss have some time to work in pairs on the vocabulary, trying to clarify the meaning of the keywords through discussion and comparison. | **Skills**

 L S R W

 Key vocabulary
 label, arrange, order, number, product, factor, element, permutation.

 Communicative structures
 Sentence structures related to mathematical relations, and to giving/asking information - i think … means … - do you know what … means? | ☑ Whole class
 ☐ Group work
 ☑ Pair work
 ☐ Individual work | T shows the video "Zero factorial or 0!": link (from Khan Academy). | None. |

| 5 | 20 | Employing the knowledge acquired during the previous activities to solve a problem about anagrams. Improving own problem solving skills. Presenting results to peers. Comparing own results with those of peers. Giving opinions/comments. | - T asks Ss to calculate the number of possible anagrams of the word "ROME". - The solution is discussed with the whole class. - Ss discover that the problem is equivalent to finding the number of permutations of 4 elements. - T asks what is the number of anagrams of the word "SCHOOL". - Through discussion the formula for the number of permutations with repeated elements is found. | **Skills**

L **S** R **W**

Key vocabulary
anagram, permutation, letter, repetition

Communicative structures
Sentence structures related to mathematical relations, to making hypotheses, to the description of a diagram, e.g. - let's make a tree diagram to count the anagrams. | ☒ Whole class
☐ Group work
☐ Pair work
☐ Individual work | None. | Formative: T assesses the insight of the hypothesis made by Ss. T informally assesses the language used to formulate the hypotheses. |

| 6 | 10 | Discovering a recursive relation useful to calculate factorials. Creative thinking: identifying patterns. Presenting results. Comparing own results with those of peers. Giving opinions/comments. | - Ss form pairs. - T asks Ss to write the factorials of the numbers from 1 to 6 in the form of products (e.g. 1!=1, 2!= 2*1, 3!= 3*2*1, ...). - Working in pairs Ss have to examine these equations and try to find a common pattern. - After 5 minutes of pair work, the whole class discusses the matter. - T directs the discussion until the recursive relation n!=n*(n-1)! Is found. - The recursive relation is then used to simplify the formula for the number of permutations of n elements in k places, and to rewrite it in terms of factorials as $D\_n,k=n!/(n-k)!$ | **Skills**

L \| S \| R \| **W**

Key vocabulary
factorial, factor, product, previous/following

Communicative structures
Sentence structures related to mathematical relations, to making hypotheses. - The first factor is always the same number that appears in the factorial. | ☑ Whole class
☐ Group work
☑ Pair work
☐ Individual work | None. | Formative: T assesses the insight of the hypothesis made by Ss. T informally assesses the language used to formulate the hypotheses. During the pair work T goes around the class supervising the work, giving advice if needed. |

| 7 | 15 | Combining the knowledge acquired during the previous activities and the knowledge about the solution of equations to solve a problem. Improving own problem solving skills. Presenting results. | - T asks Ss to use the recursive formula found in the previous activity to solve Exercise 9 of the file U1_L1_ALL1.pdf. - After 5 minutes a S is chosen to solve the problem for the whole class at the blackboard. - Ss are asked to solve Exercises 4, 6, 7, 8,10 of the same file. The ones not done in class are left as homework. | **Skills** L S R W **Key vocabulary** factorial, factor, product, division, fraction, numerator/denominator, previous/following **Communicative structures** Sentence structures related to mathematical relations, to making hypotheses, to describing steps of a solution, e.g. - I use the recursive formula at the numerator of the first fraction,.. | ☐ Whole class ☐ Group work ☐ Pair work ☐ Individual work | • U1_L1_ALL1.pdf | During the activity T goes around the class evaluating the level of comprehension of Ss (asking targeted questions if necessary). T assesses the correctness of the solution given at the blackboard and the language used to present it. |

CLIL Lesson Plan

Unit number	1	Lesson number	3	Title	Combinations and binomial coefficients

Activity	Timing	Learning Outcomes	Activity Procedure	Language	Interaction	Materials	Assessment
1	10	Reviewing the results of the previous lesson. Consolidating the knowledge acquired in the previous lesson. Self-assessment.	- T briefly reviews the main concepts examined during the previous lesson. - Ss can ask questions about the contents of the previous lesson. - The solutions of the homework are discussed.	**Skills** L S R W **Key vocabulary** factorial, factor, product, division, fraction, numerator/denominator, previous/following **Communicative structures** Sentence structures related to mathematical relations, to making hypotheses, to describing steps of a solution, e.g. - to solve the exercise... I have used the recursive relation for factorials.	☑ Whole class ☐ Group work ☐ Pair work ☐ Individual work	• U1_L1_ALL1.pdf	Formative: T assesses the solution of the problem for both content and language. Self-assessment: Ss compare their solutions to the correct ones.

| 2 | 20 | Making hypotheses. Improving own problem solving skills. Modelisation. | - T proposes the following problem: "we have two chairs labeled #1 and #2: in how many ways can we choose 4 Ss (named A, B, C, D) on the 2 chairs". - The class discusses how this problem is different from the one given in Activity 1 of Lesson 1. - T stresses that in this case a situation with S A in chair #1 and S B in chair #2 is equivalent to the situation with S B in chair #1 and S A in chair #2. - Ss are encouraged to make hypotheses, to motivate them, and to illustrate them to the rest of the class. - To clarify the problem T can call 4 Ss to the blackboard and, | **Skills**

 L \| S \| R \| **W**

 Key vocabulary
 label, arrange, order, choose.

 Communicative structures
 Sentence structures related to mathematical relations, to making and motivating hypotheses, to agreeing/disagreeing, e.g. - we cannot use the formula for permutations because... | ☑ Whole class
 ☐ Group work
 ☐ Pair work
 ☐ Individual work | None. | Formative: T assesses the insight of the hypotheses made by Ss. T informally assesses the language used to formulate the hypotheses. |

| | | | using two chairs, simulate some of the configurations described by the problem. - The problem is then solved using a tree-diagram and counting the number of permutations that involve the same elements. | | | | | |

| 3 | 15 | Employing the knowledge acquired during the previous activities to solve a problem that involves the combinations of n elements taken k at a time. Improving own problem solving skills. Presenting results. Comparing own results with those of peers. Giving opinions/comments. | - T asks Ss to use the tree-diagram method to solve the same problem of the previous activity but in the case that the chairs are 3 instead of 2. - Ss have 5 minutes to solve the problem individually. - Then Ss form pairs and discuss the solutions with their pair-mate (for about 5 minutes). - The solution is finally discussed with the whole class. - A S can be asked to solve the problem for the whole class at the blackboard. | **Skills**

L S R W

Key vocabulary
tree diagram, branch, node, path, factorial, permutation

Communicative structures
Sentence structures related to mathematical relations, to making and motivating hypotheses, to agreeing/disagreeing, e.g. - we have to count all the paths of the tree diagram that involve the same elements. | ☒ Whole class
☐ Group work
☒ Pair work
☒ Individual work | None. | During the activity T goes around the class evaluating the level of participation and comprehension of Ss (asking targeted questions if necessary). |
| 4 | 20 | Generalising the results of specific problems. Making hypotheses, Critical thinking. Checking the validity of an hypothesis. | - T asks Ss to find the general formula that describes the number of ways to take n elements k at a | **Skills**

L S R W | ☒ Whole class
☒ Group work
☐ Pair work
☐ Individual work | None. | During the activity T goes around the class evaluating the level of participation |

		Cooperation in a group work. Learning the definition of binomial coefficient and the main properties of binomial coefficients.	time. - Ss work in groups for 7 minutes. - Ss make hypothesis and discuss them with their group mates. - When 7 minutes expire T writes on the blackboard the solutions proposed by the different group. - The solution are discussed with the whole class. - The term "combination" is introduced. - The correct solution is written in the form $C\_nk = n!/((n-k)!k!) = D\_nk/k!$, highlighting that the number of combinations is equal to the number of permutations of n elements in k places divided by k! (i.e. by the number of permutations that involve the same	**Key vocabulary** Generalise, product, factor, element, permutation, combination, factorial, integer (number). **Communicative structures** Sentence structures related to mathematical relations, to making hypotheses, to the description of a diagram, e.g. - the solution is a product of … factors. - the number of permutations of n elements (in n places) is… - the factorial of a (positive) integer number is defined as…		and comprehension of Ss. Self-assessment: Ss can compare their solution to the correct one. During the discussion of the solution T assesses the hypotheses made by Ss for insight and language

				Skills					
			k elements). - The definition of binomial coefficient is introduced. - The main properties of the binomial coefficients are derived.						
5	20 [15 video+5 writing]	Understanding the main points of a short video about the properties of Pascal(Tartaglia)'s triangle and its relation to the binomial coefficients. Learning how to use binomial coefficients to calculate the coefficients of the expansion of a binomial. Identifying important data, information, and keywords. Vocabulary building.	- Ss watch a video the properties of Pascal(Tartaglia)'s triangle and its relation to the binomial coefficients. During the video Ss take notes and ask questions. - At the end of the video Ss have some time to work in pairs on the vocabulary, trying to clarify the meaning of the keywords through discussion and comparison.	**Skills** L S R W **Key vocabulary** polinomial, binomial, binomial coefficient, n-th power. **Communicative structures** Sentence structures related to mathematical relations, e.g. - the coefficient of the third term in the expansion of the n-th power of … is…	▣ Whole class ☐ Group work ▣ Pair work ☐ Individual work	T shows the youtube video "The mathematical secrets of Pascal's triangle":link (from TED-Ed). T writes on the blackboard during the complementary explanation about the relation between binomial coefficients and the coefficients in the expansion of a binomial.	None.		

| 6 | 15 | Employing the knowledge acquired during the previous activities to solve exercises that involve combinations of n elements taken k at a time and/or binomial coefficients. Improving own problem solving skills. Presenting results. Comparing own results with those of peers. Giving opinions/comments. | - T asks Ss to use the properties of binomial coefficients to solve Exercise 5 of the exercise sheet "Combinations" (file U1_L3_ALL1.pdf). -Ss have 5 minutes to solve the problem individually. - The solution is then discussed with the whole class. - In the remaining time Ss solve exercises 1, 2, 6 of the exercise sheet. - The other exercises of the exercise sheet are left as homework. | **Skills**

L S R W

Key vocabulary
binomial, binomial coefficient, combinatorics, combination. See also the "Glossary" section in the file U1_L3_ALL1.pdf.

Communicative structures
Sentence structures related to mathematical relations, e.g. -12 choose 5 (which is an alternative but common way of indicating the binomial coefficient of 12 over 5) | ▣ Whole class
☐ Group work
☐ Pair work
▣ Individual work | • U1_L3_ALL1.pdf
• U1_L3_ALL2.zip

Each S receives a printed copy of the exercise sheet "Combinations" (file U1_L3_ALL1.pdf). | During the activity T goes around the class evaluating the level of participation and comprehension of Ss (asking targeted questions if necessary). |

CLIL Lesson Plan

Unit number	1	Lesson number	4	Title	Probability - Classical definition

Activity	Timing	Learning Outcomes	Activity Procedure	Language	Interaction	Materials	Assessment
1	15	Reviewing the main topics examined during the lessons about permutations and combinations. Consolidating the knowledge acquired in the previous lesson. Correcting (some of) the exercises left as homework. Self-assessment.	- T briefly reviews the main concepts examined during the previous lesson. - Ss can ask questions about the contents of the previous lesson. - The solutions of (some of) the exercises left as homework are discussed.	**Skills** L S R W **Key vocabulary** binomial, binomial coefficient, combinatorics, combination. See also the "Glossary" section in the file U1_L3_ALL1.pdf. **Communicative structures** Sentence structures related to mathematical relations, to the description of steps of a solution e.g. - we have defined 0! as... - I started by calculating..., then I...	◙ Whole class ☐ Group work ☐ Pair work ☐ Individual work	• U1_L3_ALL1.pdf	Formative: T assesses the solution of the problem for both content and language. Self-assessment: Ss compare their solutions to the correct ones.

| 2 | 15 | Making hypotheses. Critical thinking. Checking the consistency of a hypothesis. Learning the classical definition of probability. | - T directs a discussion about probability - Ss are asked to intuitively estimate the probability of a few events (e.g. getting heads when flipping a fair coin, winning a lottery,...). - T conducts the conversation in a way that Ss come to understand that the probability of an event can be represented by a positive number less than or equal to 1. - T asks Ss what could be a good definition of probability, and the various hypotheses are discussed. - T then gives the definition of classical probability. | **Skills**

L   S   R   W

Key vocabulary
Probability, likelihood, percentage, likely, event, fair coin, fair die/dice, outcome.

Communicative structures
Sentence structures related to mathematical relations, to making hypotheses, e.g. - the probability of an event can be described by a percentage. | ☑ Whole class
☐ Group work
☐ Pair work
☐ Individual work | None. | Formative: T assesses the insight of the hypotheses made by Ss. T informally assesses the language used to formulate the hypotheses. |

| 3 | 20 | Employing the knowledge acquired during the previous lessons and activities to solve exercises that involve the calculation of probabilities Improving own problem solving skills. Presenting results. Comparing own results with those of peers. Giving opinions/comments. | - T works out two simple examples at the blackboard to prove that the tree diagram method can be used to count the favorable and the possible outcomes of an event, and therefore to calculate probabilities. - Ss have 5 minutes to solve individually Exercises 1, 2, and 3 of the exercise sheet "Probability – part 1" (file U1_L4_ALL1.pdf). - Then Ss form pairs and discuss their solutions with their pair-mate (for about 5 minutes). - The solutions are then discussed with the whole class. - Three Ss are asked to present their solutions to the class. | **Skills**

| L | S | R | W |

Key vocabulary
Probability, percentage, event, fair coin, fair die/dice, outcome.

Communicative structures
Sentence structures related to mathematical relations, to making hypotheses, to presenting result, and to giving opinions and comments e.g. - Following the definition of probability we have to calculate... - According to me the number of favorable outcomes is..." - I disagree: I think the right result is… because… | ▣ Whole class
☐ Group work
▣ Pair work
▣ Individual work | • U1_L4_ALL1.pdf
• U1_L4_ALL2.zip

Each S receives a printed copy of the exercise sheet "Probability – part 1" (file U1_L4_ALL1.pdf - editable version U1_L4_ALL2.zip). | During the activity T goes around the class evaluating the level of participation and comprehension of Ss (asking targeted questions if necessary). The solutions presented during the final discussion are then assessed for both content and language. |

| 4 | 20 | Making hypotheses. Using the knowledge acquired during the previous activities to deduce a rule that has general validity. Critical thinking. Checking the consistency of an hypothesis. Learning to calculate the probability of the complement of an event. | - T givers the definition of the complement of an event (remarking the analogy with the complement of a set in set theory). - T asks Ss to examine the exercises solved in the previous activity, and, for each one, to calculate the probability of the complement of the event considered. - The results are discussed with the whole class and the general formula for the probability of the complement of an event is deduced: $p(notE)=1-p(E)$. | **Skills** L S R W **Key vocabulary** Set, set theory, complement, Venn diagram, probability, percentage, event, fair coin, fair die/dice, outcome. **Communicative structures** Sentence structures related to mathematical relations, to making hypotheses, to presenting result, and to giving opinions and comments e.g. - Following the definition of probability we have to calculate... - According to me the number of favorable outcomes is... - I disagree: I think the right result is... because... | ▣ Whole class ☐ Group work ☐ Pair work ▣ Individual work | None. | Formative: T assesses the insight of the hypotheses made by Ss. T informally assesses the language used to formulate the hypotheses. |

| 5 | 20 [15 video+5 writing] | Understanding the main points of a short video about the "birthday problem" (i.e. what is the probability that two people in a given group share the same birthday). Understanding how the rule for the probability of the complement of an event can be useful to simplify a class of complex problems. Identifying important data, information, and keywords. Vocabulary building. | - Ss watch a video about the solution of the birthday problem. - T pauses the video and adds clarifications when needed. - During the video Ss take notes and ask questions. - At the end of the video Ss have some time to work in pairs on the vocabulary, trying to clarify the meaning of the keywords through discussion and comparison. - Ss add the keywords identified in the video to the list of words already included in the section "Glossary" of the file U1_L4_ALL1.pdf. | **Skills**

| L | S | R | W |

Key vocabulary
chance, leap year, twin, share, likely, even odds.

Communicative structures
- there is a ... chance that two people share the same birthday" - To figure out the answer you might... | ☒ Whole class
☐ Group work
☒ Pair work
☐ Individual work | T shows the youtube video "Check your intuition: the birthday problem": link (from TED-Ed). | During the pair work activity T goes around the class evaluating the level of participation and comprehension of Ss (asking targeted questions if necessary). |

| 6 | 10 | Employing the knwoledge acquired during the lesson. Learning to calculate the probability of (the complement of) simple events. Making links between the theory and the concrete situations. Learning problem solving strategies. Self-assessment. | - T gives instructions to Ss to open an online applet that challenges Ss to solve 7 simple problems about probability. - Ss work individually on the PCs available in the ICT lab. The exercises of the exercise sheet U1_L4_ALL1.pdf that were not solved in class are left as homework. | **Skills**

 | L | S | **R** | W |

 Key vocabulary
 probability, event, outcome, complement, fair die, fair coin.

 Communicative structures
 Sentence structures related to mathematical relations, to the description of steps of a solution e.g. - what is the probability that....? | ☐ Whole class
 ☐ Group work
 ☐ Pair work
 ▣ Individual work | Ss work individually on the PCs available in the ICT lab. T gives them the link to find the web-page "Simple probability – practice" (link link) from the website of Khan Academy, The applet makes sure that every S will get different exercises. | T goes around the lab offering help to better understand the tasks, and supervises the work of the Ss assessing their ability to solve the problems. |

CLIL Lesson Plan

Unit number	1	Lesson number	5	Title	Probability - Union

Activity	Timing	Learning Outcomes	Activity Procedure	Language	Interaction	Materials	Assessment
1	15	Reviewing the main topics examined during the previous lesson. Consolidating the knowledge acquired in the previous lesson. Correcting (some of) the exercises left as homework.	- T briefly reviews the main concepts examined during the previous lesson. - Ss can ask questions about the contents of the previous lesson. - The solutions of (some of) the exercises left as homework are discussed (Ss can be asked to solve the exercises at the blackboard).	**Skills** | L	S	R	W | **Key vocabulary** probability, event, outcome, complement, fair die, fair coin. See also the "Glossary" section in the file U1_L4_ALL1.pdf.	▣ Whole class ☐ Group work ☐ Pair work ☐ Individual work	• U1_L4_ALL1.pdf	Formative: T assesses Ss that correct the exercises at the blackboard for the correctness of the solution and for the language used to present it. Self assessment: Ss can compare the correct solution of the exercises to their own and evaluate their level of comprehension.

				Communicative structures Sentence structures related to mathematical relations, to the description of steps of a solution e.g. - the classical definition of probability states that... - I counted the paths of the tree diagram that represent...			

| 2 | 10 | Reviewing set theory. Recalling the definitions of the operations of union and intersection between sets, as well as the corresponding mathematical notation. Activating prior knowledge. | - T asks Ss to explain what the intersection and the union of two sets are. - T asks the S to give a few examples and to represent them with Venn diagrams at the blackboard. - The mathematical notation relative to the operations between sets is briefly reviewed. - Ss solve a simple exercise about operations with sets. | **Skills**

| L | S | R | W |

Key vocabulary
set, subset, element, intersection, union, complement, difference, Venn diagram.

Communicative structures
Sentence structures related to mathematical relations, to the description of sets, e.g. - set A contains 5 elements. - B is the subset of set A containing… - element 1 belongs to set A but also o the subset B. | ☒ Whole class
☐ Group work
☐ Pair work
☒ Individual work | None. | None. |
| 3 | 15 | Employing the knowledge acquired during the previous lessons. Calculating the probability of the union of two events. Making and discussing hypotheses. | - T asks Ss to solve three exercises that involve calculating the probability of the union of two events using tree diagrams. - The events considered in the | **Skills**

| L | S | R | W |

Key vocabulary
probability, union, coin flip, fair coin, fair die. | ☒ Whole class
☐ Group work
☐ Pair work
☒ Individual work | None. | Formative: T assesses Ss that correct the exercises at the blackboard for the correctness of the solution and for the language used to present it. Self |

exercises are mutually exclusive. A possible simple exercise is for instance: "Consider a single coin flip, and the two events A=finding heads, and B= finding tails. Find the probability of A ∪ B." - Ss have 10 minutes to solve the exercises. - The exercises are then corrected for the whole class at the blackboard: Ss can be asked to go to the blackboard to correct the exercises for their classmates. - T encourages Ss to compare the probabilities of the single events considered to the probability of their union. - Ss realise that in all the exercises the probability of the

Communicative structures
Sentence structures related to mathematical relations, e.g. - the probability of the union of A and B is equal to the probability of A plus the probability of B.

assessment: Ss can compare the correct solution of the exercises to their own and evaluate their level of comprehension.

			union is the sum of the probabilities of the single events. - At the end of the discussion T writes on the blackboard the formula P(A ∪ B) = p(A)+p(B) (formula valid only for mutually exclusive events).				

4	25	Creative thinking. Logical reasoning. Inventing and writing the text of a problem. Checking the validity of a hypothesis. Cooperating.	- Ss work in groups of 3 or 4 for 10 minutes. - T asks each group to come up with two examples (involving flipping coins or throwing dice) for which the formula found in the previous activity is not valid. The examples have to be written in the form of a text of a problem. - After 10 minutes each group passes the paper with the two problems to	**Skills** | L	S	R	W | **Key vocabulary** event, example, task, mutually exclusive. **Communicative structures** Sentence structures related to mathematical relations, to giving a task, e.g. - Calculate the probability of the union of …	▣ Whole class ▣ Group work ☐ Pair work ☐ Individual work		Peer assessment. The groups verify the correctness of the examples designed by their class mates.

the next group. - The groups solve the problems they just received. - When other 10 minutes expire, the two hardest exercises are discussed with the whole class. - T stresses that the formula found in the previous activity is valid only for mutually exclusive events.

| 5 | 15 | Understanding the main points of a short video in which the general formula for the probability of the union of two events (the addition rule) is derived using Venn diagrams. Learning the general forumla for the probability of the union of two events. Identifying important data, information, and keywords. Vocabulary building. | - Ss watch a video in which the general formula for the probability of the union of two events (the addition rule) is derived using Venn diagrams. - T pauses the video and adds clarifications when needed. - During the video Ss take notes and ask questions. | **Skills**

L S R W

Key vocabulary
event, probability, union, or, intersection, Venn diagram, set.

Communicative structures
Sentence structures related to mathematical relations, and to operations between sets, e.g. - the probability of the union of two events is… | ▣ Whole class
☐ Group work
☐ Pair work
☐ Individual work | T shows the youtube video "Addition rule for probability": link (from Khan Academy) on the IWB. | None. |
| 6 | 20 | Employing the knowledge acquired during the previous activities to solve exercises that involve calculating the probability of the union of two events. Improving own problem solving skills. Presenting results to peers. Comparing own results with those | - T shows how using a 2 way table is a convenient way to solve exercises that involve the results obtained by throwing 2 dice. - T asks Ss to use this method to solve Exercises 3, 4, and 6 of the exercise sheet | **Skills**

L S R W

Key vocabulary
event, probability, union, intersection, Venn diagram, set. See also the "Glossary" section in U1_L5_ALL1.pdf | ▣ Whole class
☐ Group work
▣ Pair work
▣ Individual work | • U1_L5_ALL1.pdf
• U1_L5_ALL2.zip

Each S receives a printed copy of the exercise sheet "Probability – part 2" (file U1_L5_ALL1.pdf - editable version U1_L5_ALL2.zip). | During the activity T goes around the class evaluating the level of participation and comprehension of Ss (asking targeted questions if necessary). |

| | | of peers. Giving opinions/comments. | "Probability – part 2" (file U1_L5_ALL1.pdf). - After 5 minutes Ss form pairs and discuss the solution. - After 5 more minutes the solution is discussed with the whole class. - In the remaining time Ss solve exercises 1 and 2 from the exercise sheet. - The solution of these two exercises is then discussed with the whole class. - The other exercises of the exercise sheet are left as homework. | **Communicative structures** Sentence structures related to mathematical relations, to sharing and discussing results, to giving/receiving advice, e.g. - My result is ...What is yours? - I think you made a mistake: look at... | | | |

CLIL Lesson Plan

Unit number		1	Lesson number		6	Title	Probability - Conditional probability

Activity	Timing	Learning Outcomes	Activity Procedure	Language	Interaction	Materials	Assessment
1	15	Reviewing the main topics examined during the previous lesson. Consolidating the knowledge acquired in the previous lesson. Correcting (some of) the exercises left as homework.	- T briefly reviews the main concepts examined during the previous lesson. - Ss can ask questions about the contents of the previous lesson. - The solutions of (some of) the exercises left as homework are discussed (Ss can be asked to solve the exercises at the blackboard).	**Skills** `L` `S` R W **Key vocabulary** event, probability, union, intersection, Venn diagram, set. See also the "Glossary" section in U1_L5_ALL1.pdf **Communicative structures** Sentence structures related to mathematical relations, to the description of steps of a solution e.g. - I counted the paths of the tree diagram that represent...	◉ Whole class ☐ Group work ☐ Pair work ☐ Individual work	• U1_L5_ALL1.pdf	Formative: T assesses Ss that correct the exercises at the blackboard for the correctness of the solution and for the language used to present it. Self assessment: Ss can compare the correct solution of the exercises to their own and evaluate their level of comprehension.

| 2 | 15 | Learning what conditional probability is. Understanding the concrete applications of conditional probability. | - T briefly introduces the concept of conditional probability and gives examples. - A simple example is worked out at the blackboard: T shows how problems involving conditional probability can be solved using tree diagrams. - Ss take notes and ask questions or clarifications. | **Skills**

 L · S · R · W

 Key vocabulary
 event, probability, conditional, given, tree diagram, branch, node.

 Communicative structures
 Sentence structures related to mathematical relations, to the description of steps of a solution e.g. - the probability of event A given event B is ... - we can delete the branch corresponding to... | ▣ Whole class
 ☐ Group work
 ☐ Pair work
 ☐ Individual work | None. | None. |

| 3 | 15 | Employing the knowledge acquired during the previous activities to solve exercises that involve calculating conditional probabilities Improving own problem solving skills. Presenting results to peers. Comparing own results with those of peers. Giving opinions/comments. Peer- and self-assessment. | - T asks Ss to use the tree diagram method to solve Exercises 1 and 2 of the exercise sheet "Probability – part 3" (file U1_L6_ALL1.pdf). - After 5 minutes Ss form pairs and discuss the solution. - After 5 more minutes the solution is discussed with the whole class: two Ss are asked to present the solution of the exercises at the blackboard to the rest of the class. | **Skills**

 L S R W

 Key vocabulary
 event, probability, conditional, given, tree diagram, branch, node, coin flip, heads/tails.

 Communicative structures
 Sentence structures related to mathematical relations, to the description of steps of a solution e.g. - the probability of ... given that at least one coin landed on ,,, | ▣ Whole class
 ☐ Group work
 ▣ Pair work
 ▣ Individual work | • U1_L6_ALL1.pdf
 • U1_L6_ALL2.zip

 Each S receives a printed copy of the exercise sheet "Probability – part 3" (file U1_L6_ALL1.pdf - editable version U1_L6_ALL2.zip). | During the activity T goes around the class evaluating the level of participation and comprehension of Ss (asking targeted questions if necessary). Peer- and self-assessment: Ss compare their solutions to those of their classmates and to the correct ones. |

| 4 | 20 | Employing the knowledge acquired during the previous activities to solve exercises that involve calculating conditional probabilities. Improving own problem solving skills. Creative thinking. Presenting results to peers. Comparing own results with those of peers. Giving opinions/comments. | - T asks Ss to use the two-way table method to solve Exercises 3 and 4 of U1_L6_ALL1.pdf. - The second exercise involves 3 dice and requires some creative thinking to be solved. - After 10 minutes Ss form pairs and discuss the solution. - After 5 more minutes the solution is discussed with the whole class: two Ss are asked to present the solution of the exercises at the blackboard to the rest of the class. | **Skills** L S R W **Key vocabulary** event, probability, conditional, given, two-way table, cell, column, row. **Communicative structures** Sentence structures related to mathematical relations, to the description of steps of a solution e.g. - we must consider the row/column/cell of the table corresponding to... | ▣ Whole class ☐ Group work ▣ Pair work ▣ Individual work | • U1_L6_ALL1.pdf | During the activity T goes around the class evaluating the level of participation and comprehension of Ss (asking targeted questions if necessary). Peer- and self-assessment: Ss compare their solutions to those of their classmates and to the correct ones. |

| 5 | 15 | Generalising the results of specific problems. Making hypotheses, Critical thinking. Checking the validity of a hypothesis. | - T asks Ss to analyse the solutions of the exercises solved in the previous two activities and try to find the general formula for the conditional probability of an event A given that an event B has occurred. - T directs the discussion until the formula $p(A|B)=p(A\cap B)/p(B)$ is found | **Skills** L S R W **Key vocabulary** event, probability, conditional, given, formula, intersection, numerator, denominator, tree diagram, path, node, branch, two-way table, cell, column, row. **Communicative structures** Sentence structures related to mathematical relations, to describing steps of a solution, to the description of diagrams and tables, e.g. - the numerator represents the numer of paths corresponding to…The denominator is instead… | ▣ Whole class ☐ Group work ☐ Pair work ☐ Individual work | None. | During the discussion of the solution T assesses the hypotheses made by Ss for insight and language |

| 6 | 5 | Visualisinig the concept of conditional probability. Understanding the connection between theory and a concrete example. | - Ss use an online simulator that visualises a situation that can be described in terms of conditional probability. - Ss can tune the parameters of the simulator (e.g. changing the probability of the single events) and visualise how this affects the value of the conditional probability. | **Skills**

L   S   **R**   **W**

Key vocabulary
event, probability, conditional, given, shelf.

Communicative structures
Sentence structures related to mathematical relations, and to describing the context of the problem, e.g. - If we have a ball and we know it hit the red shelf, there's a 50.0% chance it also hit the blue shelf. | ☐ Whole class
☐ Group work
☐ Pair work
☐ Individual work | Ss work individually on the PCs available in the ICT lab. T gives them the link to find the web-page "Conditional probability – A Visual explanation by Victor Powell for Setosa" (link link) | None. |

| 7 | 15 | Employing the knowledge acquired during the previous activities to demonstrate the validity of an equation. Improving own problem solving skills. Creative thinking. Improving abstraction ability. Presenting results to peers. Giving opinions/comments. | - T asks Ss to work in groups of 3 or 4 to solve Exercises 6 and 7 of U1_L6_ALL1.pdf. - The exercises require to prove the validity of a formula in two different ways. - Exercise 6 asks Ss to design an example that can be used to test the validity of the formula. - Exercise 7 asks for a real demonstration. - The solution is discussed with the whole class: two Ss are asked to present the solution of the exercises at the blackboard to the rest of the class. - The exercises of the exercise sheet U1_L6_ALL1.pdf that were not solved in class are left as homework. | **Skills**

 L S R W

 Key vocabulary prove, event, probability, conditional, given, shelf.

 Communicative structures Sentence structures related to mathematical relations, giving suggestions/advice, and to describing an experiment, e.g. - We could think of an example that involves flipping 3 coins. | ▣ Whole class
 ▣ Group work
 ☐ Pair work
 ☐ Individual work | • U1_L6_ALL1.pdf | During the activity T goes around the class evaluating the level of participation and comprehension of Ss (giving advice to the groups if needed). Self-assessment: Ss compare their solutions to the correct ones. |

CLIL Lesson Plan

Unit number	1	Lesson number	7	Title	Probability - Independent events

Activity	Timing	Learning Outcomes	Activity Procedure	Language	Interaction	Materials	Assessment
1	15	Reviewing the main topics examined during the previous lesson. Consolidating the knowledge acquired in the previous lesson. Correcting (some of) the exercises left as homework. Self-assessment.	- T briefly reviews the main concepts examined during the previous lesson. - Ss can ask questions about the contents of the previous lesson. - The solutions of (some of) the exercises left as homework are discussed (Ss can be asked to solve the exercises at the blackboard).	**Skills** ☑ L ☑ S ☐ R ☐ W **Key vocabulary** event, probability, conditional, given, union, intersection. **Communicative structures** Sentence structures related to mathematical relations, giving suggestions/advice, and to describing an experiment, e.g. - To demonstrate the equation I have used the Venn diagram representation.	☒ Whole class ☐ Group work ☐ Pair work ☒ Individual work	• U1_L6_ALL1.pdf	Formative: T assesses Ss that correct the exercises at the blackboard on the correctness of the solution and on the language used to present it. Self assessment: Ss can compare the correct solution of the exercises to their own and evaluate their level of comprehension.
2	20	- Using the	- T writes at the				During the

knowledge about conditional probability to derive the "product rule": a focmula that describes the probability of the intersection of two events as a function of the conditional probability and of the probability of one of the two events. - Manipulating a mathematical expression. - Cooperating. - Organising the group work.	blackboard the formula for the conditional probability p(A|B) and asks Ss to manipulate it in order to find p(A∩B). - p(A∩B) is derived by Ss starting from p(B|A), - The product rule is derived in the form p(A∩B) = p(A|B)p(B) = p(B|A)p(A). - Working in groups of 3/4, Ss check the validity of the formula by examining some of the exercises about conditional probability solved in the previous lesson or as homework. Ss use tree diagrams or two way tables to calculate the probability of the intersection and compare it to the product p(A∩B) = p(A|B)p(B) using the values	**Skills** L S R W **Key vocabulary** event, probability, conditional, given, union, intersection, product rule. **Communicative structures** Sentence structures related to mathematical relations, and to describing tree diagrams and two-way tables, e.g. - To obtain p(A∩B) we have to manipulate the formula for p(A|B).	☑ Whole class ☑ Group work ☐ Pair work ☐ Individual work

None.	activity T goes around the class evaluating the level of participation and comprehension of Ss (giving advice to the groups if needed). Self-assessment: Ss compare their solutions to the correct ones.

				Skills		Whole class	T shows the youtube video "Compound probability of independent events": link (from Khan Academy) on the IWB.	None.
				previously calculated.				
3	15	Understanding the main points of a short video about independent events. Identifying important data, information, and keywords. Vocabulary building.	- Ss watch a video in which the probability of the intersection of two or more independent events is calculated. - The video gives also a quick informal definition of independent events. - T pauses the video and adds clarifications when needed. - During the video Ss take notes and ask questions.	**Skills** L S R W **Key vocabulary** event, probability, coin flip, product rule, equally likely, possibility, outcome. **Communicative structures** Sentence structures related to mathematical relations, and to the description of steps of a process , e.g. - In order to calculate the probability of..., first we.... then...	▣ Whole class ☐ Group work ☐ Pair work ☐ Individual work	T shows the youtube video "Compound probability of independent events": link (from Khan Academy) on the IWB.	None.

| 4 | 20 | Employing the knowledge acquired during the previous lessons to solve problems about the probability of dependent and independent events. Learning to determine whether two events are independent or not. Improving own problem solving skills. Creative thinking. Presenting results to peers. Giving opinions/comments. Peer- and self-assessment. | - T asks Ss to solve individually exercises 1, 2, 3 of the exercise sheet "Probability – part 4" (file U1_L7_ALL1.pdf). - After 5 minutes Ss form pairs and discuss the solutions with their pair mate. - After 5 more minutes the solutions are discussed with the whole class: three Ss are asked to present the solution of the exercises at the blackboard to the rest of the class. | **Skills**

L S R W

Key vocabulary
event, probability, throw, die/dice, cell, row, column.

Communicative structures
Sentence structures related to mathematical relations, to describing tree diagrams and two-way tables, to presenting results, and to describing steps of a solution e.g. - To obtain p(A∩B) we need to count the cells corresponding to… - My result for the conditional probability of A given B is… | ▣ Whole class
☐ Group work
▣ Pair work
▣ Individual work | • U1_L7_ALL1.pdf
• U1_L7_ALL2.zip

Each S receives a printed copy of the document "Probability – part 4" (file U1_L7_ALL1.pdf, editable version U1_L7_ALL2.zip). | During the activity T goes around the class evaluating the level of participation and comprehension of Ss (giving advice to the groups if needed). Peer- and self-assessment: Ss compare their solutions to the those of their classmates and to the correct ones. |

5	10	Making hypotheses. Re-elaborating information. Learning the definition of independent events	- T invites Ss to examine the results of the exercises solved in the previous activity. - T directs the discussion until Ss grasp the concept that for two independent events A and B the conditional probability of A given B is equal to the probability of A. - The simplified form of the "product rule" valid for independent events is derived.	**Skills**	☑ Whole class ☐ Group work ☐ Pair work ☐ Individual work	None.	Formative: T assesses the insight of the hypotheses made by Ss T informally assesses the language used to formulate the hypotheses.

Skills

L	S	R	W
■	■		

Key vocabulary
event, probability, conditional, given, union, intersection, product rule, simplify.

Communicative structures
Sentence structures related to mathematical relations, to making hypotheses, to discussing results e.g. - we could substitute p(A) to p(A|B) in the formula for p(A∩B).

| 6 | 10 | Employing the knowledge acquired during the previous activities to solve problems about the probability of independent events. Improving own problem solving skills. | - T asks Ss to solve exercises 4 and 10 of the exercise sheet "Probability – part 4" (file U1_L7_ALL1.pdf).
 - The remaining exercises of the exercise sheet are left as homework | **Skills**

 L S **R** **W**

 Key vocabulary
 event, probability, independent, see also the "Glossary" section in file U1_L7_ALL1.pdf

 Communicative structures
 Sentence structures related to mathematical relations, and to asking questions/giving tasks e.g. - Does he have more chances to… or to…? | ☐ Whole class
 ☐ Group work
 ☐ Pair work
 ☒ Individual work | • U1_L7_ALL1.pdf
 • U1_L7_ALL2.zip | During the activity T goes around the class supervising the work of Ss and giving advice if needed. |

| 7 | 10 | Understanding the main points of a short video about the solution of a problem involving the probability of independent events. Identifying important data, information, and keywords. Self-assessment. Vocabulary building. Reasoning on different strategies of solution. | - Ss watch a video about the solution of exercise 10 of file U1_L7_ALL1.pdf - During the video Ss take notes, ask questions, and correct their own solution of the exercise. | **Skills**

 L S R W

 Key vocabulary event, probability, independent, see also the "Glossary" section in file U1_L7_ALL1.pdf.

 Communicative structures - The probability of … is… | ▣ Whole class
 ☐ Group work
 ☐ Pair work
 ☐ Individual work | • U1_L7_ALL1.pdf

 T shows the youtube video "Three-pointer vs free-throw probability": link (from Khan Academy) on the IWB. | Self assessment: Ss compare their solution to the correct one and compare their solution strategy to an alternative one. |

CLIL Lesson Plan

Unit number	1	Lesson number	8	Title	Probability - Total probability

Activity	Timing	Learning Outcomes	Activity Procedure	Language	Interaction	Materials	Assessment
1	15	Reviewing the main topics examined during the previous lesson. Consolidating the knowledge acquired in the previous lesson. Correcting (some of) the exercises left as homework.	- T briefly reviews the main concepts examined during the previous lesson. - Ss can ask questions about the contents of the previous lesson. - The solutions of (some of) the exercises left as homework are discussed (Ss can be asked to solve the exercises at the blackboard).	**Skills** ☑ L ☑ S ☐ R ☐ W **Key vocabulary** event, probability, conditional, given, union, intersection, dependent. **Communicative structures** Sentence structures related to mathematical relations, giving suggestions/advice, and to describing steps of a solution, e.g. - My result for... is...	◉ Whole class ☐ Group work ☐ Pair work ◉ Individual work	• U1_L7_ALL1.pdf	Formative: T assesses Ss that correct the exercises at the blackboard on the correctness of the solution and on the language used to present it. Self assessment: Ss can compare the correct solution of the exercises to their own and evaluate their level of comprehension.

| 2 | 25 | Using tree diagrams to solve a problem about the probability of a compound event. Learning to use weighted tree-diagrams. Making hypotheses. Discussing hypotheses. Giving opinions/comments. Taking notes. Identifying relevant information and concepts. | - The asks Ss to solve Exercise 1 in the exercise sheet "Probability – part 5" (file U1_L8_ALL1.pdf), - The possible ways to solve the exercise are discussed with the whole class. - A tree-diagram with weighted branches is built and the solution of the problem is found. - T guides Ss to re-discover the rule for the probability of the complement event, the product rule, and the rule for the probability of the union of two or more events, from the analysis of the nodes, of the branches and of the different paths of the weighted tree diagram. | **Skills**

| L | S | R | W |

Key vocabulary
event, probability, conditional, given, union, intersection, product rule, node, branch, path.

Communicative structures
Sentence sftructures related to mathematical relations, and to describing tree diagrams, e.g. - to find the probability of the event W we have to sum the probabilities of all the paths that lead to W. | ☑ Whole class
☐ Group work
☐ Pair work
☑ Individual work | • U1_L8_ALL1.pdf
• U1_L8_ALL2.zip

Each S receives a printed copy of the exercise sheet "Probability – part 5" (file U1_L8_ALL1.pdf - editable version U1_L8_ALL2.zip). | Formative: T assesses Ss on the insight and language of their hypotheses. |

3	10	Reading aloud. Understanding a short text. Identifying relevant information and instructions.	- Some Ss are chosen at random to read the text of the tasks of the online activity "Tree diagrams and conditional probability" from Khan Academy. - Difficult words are discussed with T and the class.	**Skills**	☑ Whole class	Ss use the PCs available in the ICT lab. The activity "Tree diagrams and conditional probability" can be found at the link link	T assesses the reading skills of Ss.

Skills

L	S	R	W

Key vocabulary
event, probability, conditional, given, union, intersection, product rule, node, branch, path, screen, trigger, (forbidden) item, patient, disease, positive/negative.

Communicative structures
Sentence structures related to mathematical relations, to gibing instructions, and to describing tree diagrams, e.g. - suppose that,,," "what is the probabilty that…?

☐ Group work
☐ Pair work
☑ Individual work

| 4 | 25 | Using the knowledge acquired in the previous lessons and activities. Interdisciplinary links. Understanding the concept of false-positive in a medical exam. Understanding and following written instructions. Identifying important information. Self assessment. | - Ss work in pairs to solve a guided complex problem about conditional probability following the written instructions. - Ss solve the problem proposed at the end of the activity. | **Skills**

| L | S | **R** | **W** |

Key vocabulary
event, probability, conditional, given, union, intersection, product rule, node, branch, path, screen, trigger, (forbidden) item, patient, disease, positive/negative.

Communicative structures
Sentence structures related to mathematical relations, to giving opinions/comments, and to describing tree diagrams, e.g. - we should multiply the probabilities corresponding to the branches. | ☐ Whole class
☐ Group work
☑ Pair work
☐ Individual work | Each pair of Ss uses one of the PCs available in the ICT lab. The activity "Tree diagrams and conditional probability" involving the guided problem and the final exercise can be found at the link link | During the activity T goes around the lab supervising the work of the pairs and assessing the level of participation and comprehension. The online activity is interactive and gives immediate feedback to Ss that can use that to self'assess their comprehension. |

				Skills			
5	10	Learning the law of total probability. Interpreting and generalising results. Taking notes.	- T invites Ss to examine the solutions of the exercises solved in the previous activity and in activity 2. - T shows that the results are consistent with the formula of total probability. - The formula of total probability is presented and explained.	**Skills** L S R W (L, S, W filled) **Key vocabulary** event, probability, conditional, given, union, intersection, product rule, node, branch. **Communicative structures** Sentence structures related to mathematical relations, to giving opinions/comments, and to describing tree diagrams, e.g. - the law of total probability expresses the total probability of …	▣ Whole class ☐ Group work ☐ Pair work ☐ Individual work	None.	None.
6	15	Employing the knowledge acquired during the previous activities to solve problems involving weighted tree diagrams. Improving own	- T asks Ss to work in groups of 3 or 4 to solve Exercises 3 and 4 of U1_L8_ALL1.pdf. - The solution is discussed with	**Skills** L S R W (S, R filled)	☐ Whole class ▣ Group work ☐ Pair work ☐ Individual work	• U1_L8_ALL1.pdf	During the activity T goes around the class evaluating the level of participation and

improving own problem solving skills. Presenting results to peers. Giving opinions/comments.

discussed with the whole class: two Ss are asked to present the solution of the exercises at the blackboard to the rest of the class. - The exercises of the exercise sheet U1_L8_ALL1.pdf that were not solved in class are left as homework.

Key vocabulary
event, probability, conditional, given, union, intersection, product rule, node, branch. See also the "Glossary" section in file U1_L8_ALL1.pdf.

Communicative structures
Sentence structures related to mathematical relations, to giving opinions/comments, and to describing tree diagrams, e.g. - the first node of the tree diagram must … branches. - The weight of this branch is the conditional probability of…

and comprehension of Ss (giving advice to the groups if needed). Self-assessment: Ss compare their solutions to the correct ones.

CLIL Lesson Plan

Unit number	1	Lesson number	9	Title	Probability - Frequentist interpretation

Activity	Timing	Learning Outcomes	Activity Procedure	Language	Interaction	Materials	Assessment
1	10	Reviewing the main topics examined during the previous lesson. Consolidating the knowledge acquired in the previous lesson. Correcting some of the exercises left as homework. Self-assessment.	- T briefly reviews the main concepts examined during the previous lesson. -Ss can ask questions about the contents of the previous lesson. - Ss can give comments and opinions about the previous lesson. - The solutions of (some of) the exercises left as homework are discussed (Ss can be asked to solve the exercises at the blackboard)	**Skills** | L	S	R	W | **Key vocabulary** event, probability, conditional, given, union, intersection, product rule, node, branch. See also the "Glossary" section in file U1_L8_ALL1.pdf	☑ Whole class ☐ Group work ☐ Pair work ☑ Individual work	• U1_L8_ALL1.pdf	Formative: T assesses Ss that solve the exercises at the blackboard on the correctness of the solution and on the language used to present it. Self assessment: Ss can compare the correct solution of the exercises to their own and evaluate their level of comprehension.

blackboard).

Communicative structures
Sentence structures related to mathematical relations, to giving opinions/comments, and to describing tree diagrams, e.g.
- to solve the exercise i have used a weighted tree diagram. - the weight of this branch is...

| 2 | 15 | Learning that probability can be studied using different approaches. Learning the motivations that make the classical theory of probability fail to solve certain problems. | - T explains the frequentist definition of probability and gives examples of the fields of study in which it is useful. - T gives examples of problems for which the classical approach to probability fails to find a solution. - Ss take notes, ask questions, and answer T's questions. - To clarify the relation between the classical approach and the frequentist approach T uses two interactive spreadsheets that simulate a large number of dice rolls (included in file U1_L9_ALL1.zip). | **Skills**

L S R **W**

Key vocabulary
event, probability, interpretation, approach, (relative) frequency, frequentist, limit, infinity, infinite, fair/unfair, die, coin.

Communicative structures
Sentence structures related to mathematical relations and definitions, and to describing examples e.g. - In the frequentist interpretation the probability of an event is defined as… | ▣ Whole class
☐ Group work
☐ Pair work
☐ Individual work | • U1_L9_ALL1.zip

T shows thet two interactive spreadsheets included in U1_L9_ALL1.zip (courtesy of Luciano Cappello – luciano.cappello@unitn.it). | None. |

| 3 | 25 | Having a better understanding of the law of large numbers. Learning about the gambler's fallacy. Proving that the outcomes of a coin flip does not influence the outcome of the next one (proving that two consecutive coin flipx/dice rolls) are independent events. Critical thinking. Identifying important concepts and information. Taking notes. | - T explains the link between probability and the law of large numbers. - T runs a video on the IWB about the gambler's fallacy. - T asks Ss to prove that two consecutive coin flips are independent events. - During the first part of the activity Ss take notes and can ask questions. - When they have to prove the fact that two consecutive coin flips are independent events, Ss work in pairs. | **Skills**

◼ L ◼ S ◼ R ◼ W

Key vocabulary
event, probability, interpretation, approach, (relative) frequency, frequentist, limit, infinity, infinte, law, large number, tend, gambler, fallacy.

Communicative structures
Sentence structures related to mathematical relations and definitions, and to describing examples e.g. - the law of large numbers states that... - the gambler's fallacy is the mistaken belief that... | ◼ Whole class
☐ Group work
◼ Pair work
☐ Individual work | T runs on the IWB the youtube video "Critical Thinking Part 5: The Gambler's Fallacy" (link: link) | During the pair work activity T goes around the class supervising the work of the pairs, giving advice if needed. |

| 4 | 45 | Employing the knowledge acquired during the previous lessons to solve problems that cover most of the topics of the unit. Tackling a question taken from the Italian scientific high school exit exam of 2014. Understanding a task. Identifying important information and concepts. Improving own problem solving skills. Creative thinking. Presenting results to peers. Giving opinions/comments. | - T asks Ss to solve the exercises of the exercise sheet "Probability – part 6" (file U1_L9_ALL1.pdf). - After 20 minutes Ss form pairs to discuss and complete the solutions with their pair mate. - After 10 more minutes the solutions are discussed with the whole class: Ss are asked to present the solution of the exercises at the blackboard to the rest of the class. | **Skills**

 L S R W

 Key vocabulary choose, arrange, count, permutation, combination, taken … at a time, event, probability, conditional, given, union, intersection, product rule, node, branch, two-way table, row, column, cell. See also the "Glossary" section in file U1_L9_ALL1.pdf

 Communicative structures Sentence structures related to mathematical relations, to giving opinions/comments, and to describing tree diagrams. - The number of combinations results… - Do our results match? | ☑ Whole class
 ☐ Group work
 ☑ Pair work
 ☑ Individual work | • U1_L9_ALL2.pdf
 • U1_L9_ALL3.zip

 Each S receives a printed copy of the document "Probability – part 6" (file U1_L9_ALL2.pdf - editable version U1_L9_ALL3.zip). | During the activity T goes around the class evaluating the level of participation and comprehension of Ss (giving advice if needed). Peer- and self-assessment: Ss compare their solutions to the ones of their classmates and to the correct ones. |

| 5 | 5 | Summarising the key concepts analysed in this unit. Self-assessment. Giving opinions and comments on the lessons. | - T summarises the key topics examined in the lessons about combinatorics and probability and gives information about the unit test that will take place during the next lesson. - Ss can ask questions and clarifications (also about the final test), | **Skills**

| L | S | R | W |

Key vocabulary
permutation, combination, probability, conditional, given, union, intersection, addition/product rule, node, branch, two-way table, row, column, cell. See also the "Glossary" section in file U1_L9_ALL1.pdf.

Communicative structures
Sentence structures related to mathematical relations, to giving information/opinions/ comments, e.g. - the final test lasts… minutes. | ▣ Whole class
☐ Group work
☐ Pair work
☐ Individual work | None. | None. |

CLIL Lesson Plan

Unit number		1	**Lesson number**		10	**Title**	Final test + discussion

Activity	Timing	Learning Outcomes	Activity Procedure	Language	Interaction	Materials	Assessment
1	60	Understanding the text and the requests of the test. Employing the knowledge acquired during this unit to solve exercises of various difficulty levels. Tackling a question taken from the Italian scientific high school exit exam of 2012. Graph interpretation. Creative thinking. Problem solving ability.	- Ss employ the knowledge acquired during this unit to solve five exercises of various difficulty levels. - T hands out the test and reads the questions out loud, making sure that all Ss have understood the tasks.	**Skills** L　S　R　W **Key vocabulary** Scientific terms used in the rest of the module. **Communicative structures** Sentence structures related to giving instruction, and necessary to answer a question and describing steps of a procedure, e.g. - you have 60'to complete the test.	☐ Whole class ☐ Group work ☐ Pair work ☑ Individual work	• U1_L10_ALL1.pdf • U1_L10_ALL2.zip Each S receives a printed copy of file U1_L10_ALL1.pdf (editable version U1_L10_ALL2.zip).	Performance evaluation.

| 2 | 30 | Understanding the mistakes done in the test. Evaluating own work. Reflecting on different solution strategies. | - T works out the correction of the various exercises at the blackboard. - Ss ask questions and propose alternative solutions. - The solutions of the exercises are discussed. | **Skills**

| L | S | R | W |

Key vocabulary
Scientific terms used in the rest of the module.

Communicative structures
Sentence structures related to mathematical relations, to the descriptions of (steps of) a process, and to proposing alternatives, e.g. - I have solved that exercise in a different way:... | ▣ Whole class
☐ Group work
☐ Pair work
☐ Individual work | • U1_L10_ALL1.pdf | Self-assessment: Ss can compare their solutions to the correct ones and assess their level of comprehension. |

| 3 | 10 | Having an overview of the topics examined during the CLIL module. Giving opinions and comments. | - T briefly reviews the topics covered during the module. - Ss give opinions and comments about the topics (e.g. interesting/not interesting), and/or about the lessons. | **Skills**

| L | S | R | W |

Key vocabulary
Scientific terms used in the rest of the module.

Communicative structures
Sentence structures related to giving opinions/comments, e.g. - I think that the topic of this module is... - I believe that thte lesson about ... was... | ☒ Whole class
☐ Group work
☐ Pair work
☐ Individual work | None. | None. |