Doppler effect and supersonic speed.

Daniele Taufer (daniele.taufer@unitn.it)
with minor additions by Giovanni Lombardi (giovanni.lombardi@liceodavincitn.it)

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1 Doppler effect

At the beginning of 19th century the Austrian physicist Christian Doppler studied the apparent change of pitch that occurs when the source of a sound or its receiver are moving. Since from the 19th century the "world speed" has been sharply increasing, this effect can now be observed in many common situations of our lives.

The qualitative observation that stands at the core of this effect is the following: when the source and the receiver of a sound signal are getting closer the pitch seems to increase, while it seems to decrease when they distance themself.

Key observation of Doppler effect :	closer $\Longrightarrow$ higher pitch.
	farther $\Longrightarrow$ lower pitch.

We start by studying the case when the source of sound is fixed and the receiver is moving towards this source.

1.1 Moving observer

The key idea is that the detected λ is constant in this setting and what changes is actually the sonar wave speed: it seems "faster" to the observer because he is moving towards the source.

Let v_o be the observer speed and v the speed of sonar waves. The apparent speed of the wave (from the observer point of view) will therefore be $v + v_o$, hence the detected frequency f' will be

$$f' = \frac{v + v_o}{\lambda} = \frac{v}{\lambda} + \frac{v_o}{\lambda}$$

and since $\lambda = \frac{v}{f}$ (where f is the actual frequency of the wave) we finally obtain the formula for the detected frequency.

$f' = f\left(1 + \frac{v_o}{v}\right)$:	frequency detected by an observer moving towards the source with speed v_o.

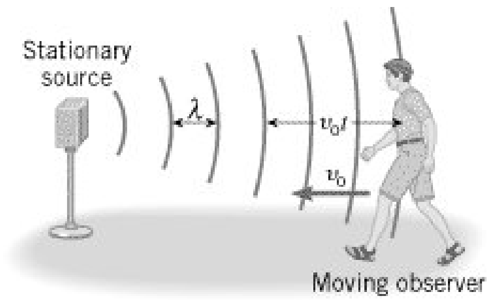

Figure 1: picture taken from http://demo.webassign.net/ebooks/cj6demo/pc/c16/read/main/c16x16_9.htm

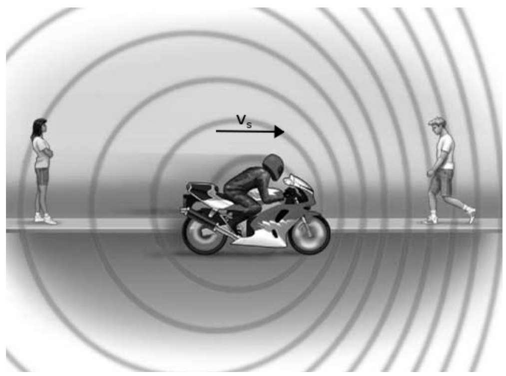

Figure 2: picture taken from https://www.slideserve.com/newman/effetto-doppler

When the observer is moving away from the source, it is sufficient to invert the sign of v_0 to get an analogous result.

$f' = f\left(1 - \frac{v_o}{v}\right)$:	frequency detected by an observer moving away from the source with speed v_o.

1.2 Moving source

Now assume that, conversely, the source is moving towards the observer, which is stationary. In this case the detected wave speed is its real speed because the observer is not moving, but the detected wavelength is different because the wave seems to be compressed, as the guy in the following figure sees.

That guy notices also another change: crests seem to arrive faster, which amounts to say that the detected frequency is higher. Conversely, the girl receives waves which seem to have longer wavelength but lower frequency, because in her case the source is getting farther.

Quantitatively, if v_s the source speed, in a period T of time the source has moved $v_s T$ forward, thus two consecutive crests will be $\lambda - v_s T$ away, which means that the detected wavelength λ' is

$$\lambda' = \lambda - v_s T = (v - v_s)T$$

Here we immediately observe that we want v to be higher than v_s, in order to have a positive λ'. This means that the source speed has to be *subsonic*, i.e. lower than the sound speed. We will discuss the opposite case in the next section. Therefore when $v > v_s$, since we said that the speed v does not change, the detected frequency will be

$$f' = \frac{v}{\lambda'} = \frac{v}{(v - v_s)T} = f\frac{v}{v - v_s} = f\frac{1}{1 - \frac{v_s}{v}}.$$

$f' = f\frac{1}{1 - \frac{v_s}{v}}$: frequency detected by an observer when a source is approaching with speed v_s.

Again, this is exactly the same for the girl apart from the sign of v_s, which has to be inverted if the source is moving away.

$f' = f\frac{1}{1 + \frac{v_s}{v}}$: frequency detected by an observer when a source is moving away with speed v_s.

Exercise: The Boeing 747 is an American wide-body commercial jet airliner and cargo aircraft, which travels with a speed of about 900 km/h.
You are now travelling on a Boeing 747 and producing a sound of frequency 3400 Hz, while the ariplane is moving towards a mountain.
(i) Ignoring any other noise (like the one created by the airplane), could you hear the sound you are producing?
(ii) Ignoring any other noise, could a person sitting on the mountain hear the sound you are producing?
(iii) The mountain reflects the waves you are producing. Ignoring any other noise, could you hear your reflected sound?

1.3 General case

The combination of the results found in the previous sections gives the general formula for the Doppler effect, when both the observer and the source are

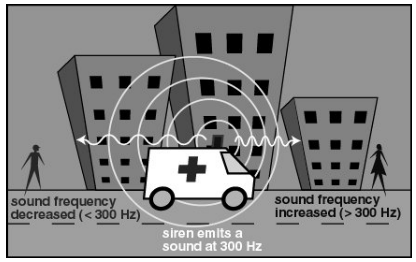

Figure 3: picture taken from `http://www.deusdiapente.net/science/bbt.php?option=red`.

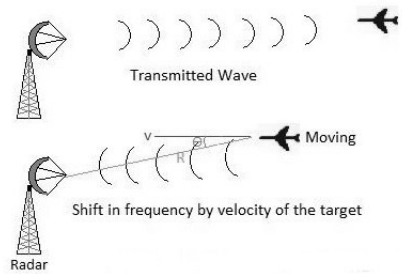

Figure 4: picture taken form `http://www.rfwireless-world.com/Tutorials/doppler-radar.html`

moving, with speeds v_o and v_s respectively.

$$f' = f\frac{1 \pm \frac{v_0}{v}}{1 \mp \frac{v_s}{v}} \text{ : general Doppler effect formula.}$$

where the signs have to be chosen according to the above discussion, i.e.:
◦ Numerator: + if the observer is moving towards the source, − otherwise.
◦ Denominator: − if the source is moving towards the observer, + otherwise.

1.4 Applications

As we saw before, almost everything in the world we live produces Doppler effects, either when the sources are moving or when the receivers are. As an instance, one of the most clear everyday experience of this effect is the siren of a passing emergency vehicle.

A common but useful application is the doppler radar, to measure speed of detected objects. Even if it does not really use sound waves but radar ones, the reasoning is the same: a radar beam is fired at a moving target and the change in the wavelength of reflected waves let the radar evaluate the target velocity.

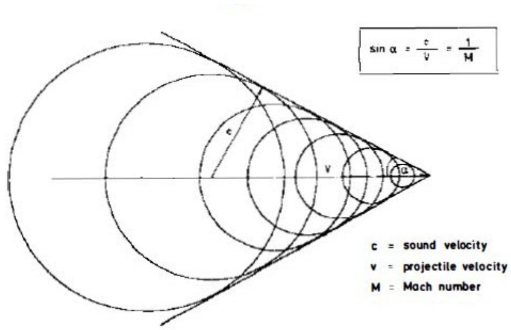

Figure 5: picture taken from `http://dropseaofulaula.blogspot.com/2013/03/ritratti-ernst-mach.html`

It has medical applications as well, especially for measuring velocities in a fluid flow (like blood).

2 Supersonic speeds

We observed above that some problems occur when the source speed exceeds the sound one. In this case we cannot use previous formulas anymore and we will just give a qualitative description of what happens. The wavefronts are generated behind the source, creating a cone which is usually called the *Mach cone*. In the following figure we see the Mach cone generated by a projectile.

The angle α is usually called *Mach angle* while the ratio between the source speed and the sound speed is called *Mach number*.

$\frac{v_s}{v}$: Mach number, where v_s is the source speed and v is the sound speed.

When you hear about a plane which flies at Mach 2.3 it simply means that its Mach number is 2.3, i.e. it is flying 2.3 times faster than sound. While you stay outside the Mach cone you do not hear any noise. Since on the surface of the cone many wavefronts pile up, you will hear a strong sound (like an explosion) when you cross this surface. This is commonly known as *sonic boom*, and you see in the below picture the exact moment at which it happens.

3 The Doppler effect in astronomy

The Doppler effect is of intense interest to astronomers who use the information about the shift in frequency of electromagnetic waves produced by moving stars in our galaxy and beyond in order to derive information about those stars and galaxies. The belief that the universe is expanding is based in part upon

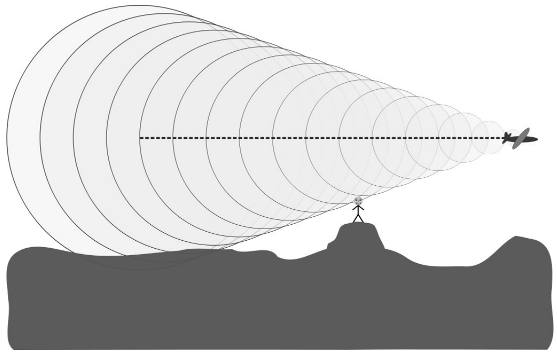

Figure 6: Image taken from https://commons.wikimedia.org/wiki/File:Sonic_boom.png and edited.

observations of electromagnetic waves emitted by stars in distant galaxies. Furthermore, specific information about stars within galaxies can be determined by application of the Doppler effect. Galaxies are clusters of stars that typically rotate about some center of mass point. Electromagnetic radiation emitted by such stars in a distant galaxy would appear to be shifted downward in frequency (a red shift) if the star is rotating in its cluster in a direction that is away from the Earth. On the other hand, there is an upward shift in frequency (a blue shift) of such observed radiation if the star is rotating in a direction that is towards the Earth. This effect is shown in Fig.7 for a system consisting of two stars (a binary system).[1]

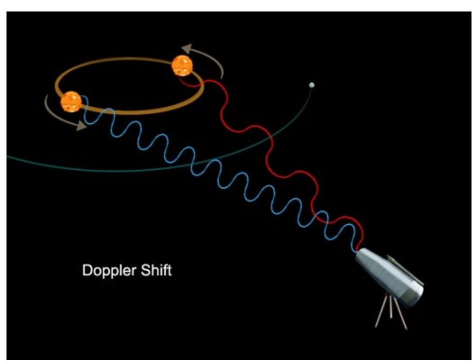

Figure 7: Doppler shift in a binary star system. Image taken from https://commons.wikimedia.org/wiki/File:Doppler_shift_caused_by_exoplanet.jpeg and edited.

[1] The text of shis section was taken from http://www.physicsclassroom.com/class/waves/Lesson-3/The-Doppler-Effect